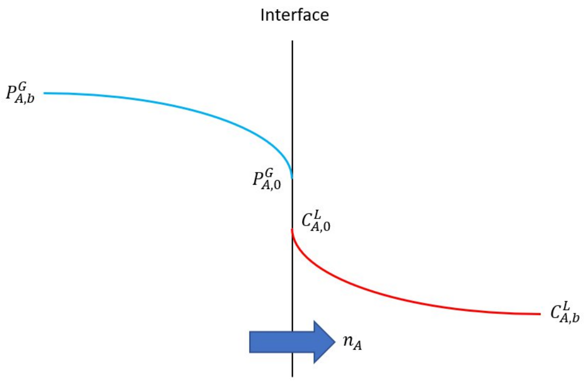
-Introduction to Transport Phenomena: Final Exam Solutions

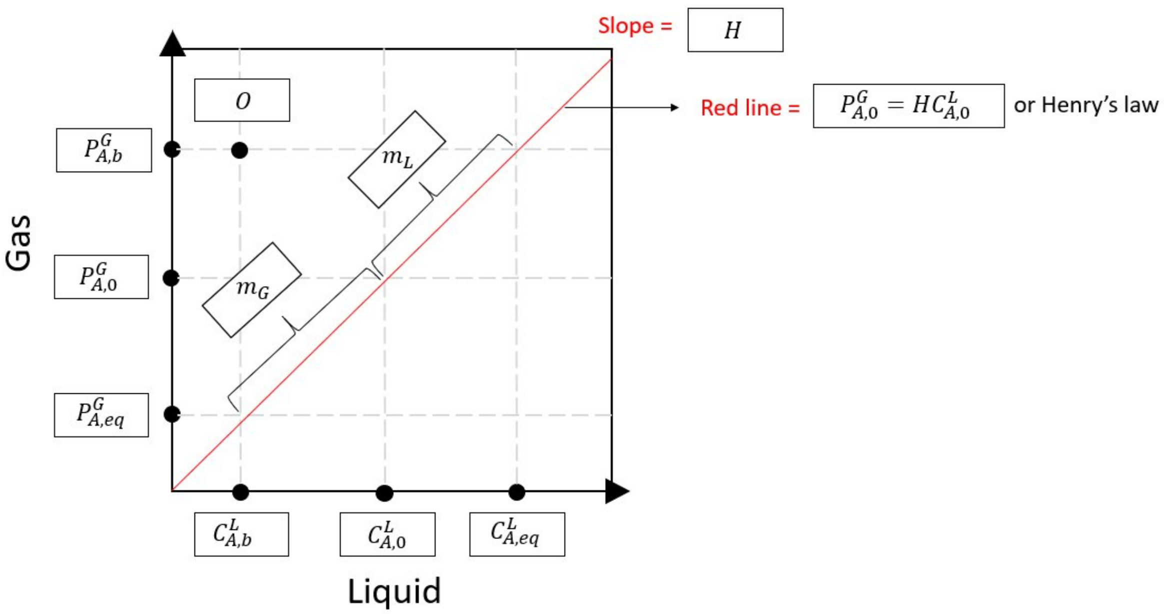
Question 1 – Solutions (15 points)

Throughout the problem, A is used for NH_3.

1) (1pt)



2) (3pts)



3) (4.5 pts) We are given:

$$C^L_{A,b} = 6.42 \cdot 10^{-2} \frac{kmol}{m^3}$$

$$H = 9.35 \cdot 10^{-3} \frac{atm \cdot m^3}{kmol}$$

$$P^G_{A,b} = 0.1\ atm$$

We have the following equations:

$$n_A = k_{L,loc}\left(C^L_{A,0} - C^L_{A,b}\right) \quad (1)$$

$$n_A = k_{G,loc}\left(P^G_{A,b} - P^G_{A,0}\right) \quad (2)$$

$$P^G_{A,0} = HC^L_{A,0} \quad (3)$$

If we divide equation 1 by equation 2, we obtain:

$$1 = \frac{k_{L,loc}}{k_{G,loc}} \cdot \frac{\left(C^L_{A,0} - C^L_{A,b}\right)}{\left(P^G_{A,b} - P^G_{A,0}\right)}$$

We can replace $P^G_{A,0}$ with $HC^L_{A,0}$

$$1 = \frac{k_{L,loc}}{k_{G,loc}} \cdot \frac{\left(C^L_{A,0} - C^L_{A,b}\right)}{\left(P^G_{A,b} - HC^L_{A,0}\right)}$$

We know everything in the equation except for the liquid concentration of NH_3 at the interface.

Doing math to solve for $C^L_{A,0}$:

$$\frac{k_{G,loc}}{k_{L,loc}}\left(P^G_{A,b} - HC^L_{A,0}\right) = C^L_{A,0} - C^L_{A,b}$$

$$\frac{k_{G,loc}}{k_{L,loc}}P^G_{A,b} + C^L_{A,b} = C^L_{A,0} + \frac{k_{G,loc}}{k_{L,loc}}HC^L_{A,0}$$

$$\frac{k_{G,loc}}{k_{L,loc}}P^G_{A,b} + C^L_{A,b} = C^L_{A,0}(1 + \frac{k_{G,loc}}{k_{L,loc}}H)$$

$$C^L_{A,0} = \frac{\frac{k_{G,loc}}{k_{L,loc}}P^G_{A,b} + C^L_{A,b}}{1 + \frac{k_{G,loc}}{k_{L,loc}}H}$$

$$C^L_{A,0} = \frac{18.88\ \frac{kmol}{m^3 \cdot atm} \cdot 0.1\ atm + 6.42 \cdot 10^{-2}\frac{kmol}{m^3}}{1 + 18.88\ \frac{kmol}{m^3 \cdot atm} \cdot 9.35 \cdot 10^{-3}\frac{atm \cdot m^3}{kmol}} = 1.66\left[\frac{kmol}{m^3}\right]$$

With Henry's law:

$$P^G_{A,0} = HC^L_{A,0} = 9.35 \cdot 10^{-3}\frac{atm \cdot m^3}{kmol} \cdot 1.66\left[\frac{kmol}{m^3}\right] = 0.0155\ [atm]$$

4)(2pts) From the equilibrium relations, we have that:

$$m_g = H = \frac{P^G_{A,0} - P^G_{A,eq}}{C^L_{A,0} - C^L_{A,b}}$$

$$m_l = H = \frac{P^G_{A,b} - P^G_{A,0}}{C^L_{A,eq} - C^L_{A,0}}$$

By rearranging:

$$P^G_{A,eq} = P^G_{A,0} - H\left(C^L_{A,0} - C^L_{A,b}\right)$$

$$C^L_{A,eq} = \frac{P^G_{A,b} - P^G_{A,0} + HC^L_{A,0}}{H}$$

Therefore:

$$P^G_{A,eq} = 0.0155\ atm - 9.35\cdot10^{-3}\frac{atm\cdot m^3}{kmol}\left(1.66\left[\frac{kmol}{m^3}\right] - 6.42\cdot10^{-2}\frac{kmol}{m^3}\right)$$

$$P^G_{A,eq} = 5.79\cdot10^{-4}[atm]$$

$$C^L_{A,eq} = \frac{0.1\ atm - 0.0155\ atm + 9.35\cdot10^{-3}\frac{atm\cdot m^3}{kmol}\cdot1.66\left[\frac{kmol}{m^3}\right]}{9.35\cdot10^{-3}\frac{atm\cdot m^3}{kmol}}$$

$$C^L_{A,eq} = 10.7\left[\frac{kmol}{m^3}\right]$$

5) (1pt) We have:

$$n_A = K_{G,\ overall}\left(P^G_{A,b} - P^G_{A,eq}\right) = 2.72\cdot10^{-4}\left[\frac{kmol}{m^2\cdot s\cdot atm}\right]\left(0.1\ atm - 5.79\cdot10^{-4}\ atm\right)$$

$$n_A = 2.70\cdot10^{-5}\left[\frac{kmol}{m^2\cdot s}\right]$$

6)(2pts) We have:

$$n_A = K_{L,overall}\left(C^L_{A,eq} - C^L_{A,b}\right)$$

$$K_{L,overall} = \frac{n_A}{\left(C^L_{A,eq} - C^L_{A,b}\right)} = \frac{2.70\cdot10^{-5}\left[\frac{kmol}{m^2\cdot s}\right]}{\left(10.7\left[\frac{kmol}{m^3}\right] - 6.42\cdot10^{-2}\frac{kmol}{m^3}\right)} = 2.54\cdot10^{-6}\left[\frac{m}{s}\right]$$

$$n_A = k_{L,loc}\left(C^L_{A,0} - C^L_{A,b}\right)$$

$$k_{L,loc} = \frac{n_A}{\left(C^L_{A,0} - C^L_{A,b}\right)} = \frac{2.70\cdot10^{-5}\left[\frac{kmol}{m^2\cdot s}\right]}{\left(1.66\left[\frac{kmol}{m^3}\right] - 6.42\cdot10^{-2}\frac{kmol}{m^3}\right)} = 1.69\cdot10^{-5}\left[\frac{m}{s}\right]$$

$$n_A = k_{G,loc}\left(P^G_{A,b} - P^G_{A,0}\right)$$

$$k_{G,loc} = \frac{n_A}{\left(P^G_{A,b} - P^G_{A,0}\right)} = \frac{2.70 \cdot 10^{-5}\left[\frac{kmol}{m^2 \cdot s}\right]}{(0.1\ atm - 0.0155\ [atm])} = 3.20 \cdot 10^{-4}\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right]$$

As a verification, we can calculate:

$$\frac{k_{G,loc}}{k_{L,loc}} = \frac{3.20 \cdot 10^{-4}\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right]}{1.69 \cdot 10^{-5}\left[\frac{m}{s}\right]} = 18.93\left[\frac{kmol}{m^3 \cdot atm}\right] \cong 18.88\left[\frac{kmol}{m^3 \cdot atm}\right]$$

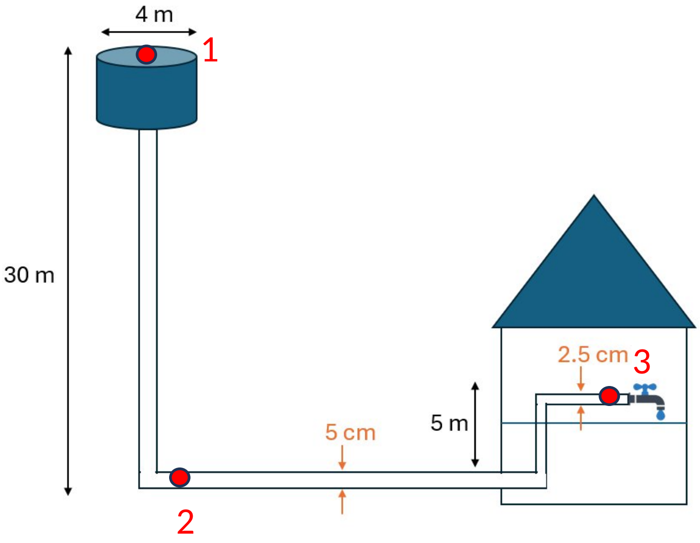
7) (1.5pts)

$$\frac{k_{L,loc}}{m_{avg} k_{G,loc}} = \frac{k_{L,loc}}{H k_{G,loc}} = \frac{1.69 \cdot 10^{-5}\left[\frac{m}{s}\right]}{9.35 \cdot 10^{-3}\frac{atm \cdot m^3}{kmol} \cdot 3.20 \cdot 10^{-4}\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right]} = 5.65$$

Since 5.65 isn't << 1 or >> 1, neither side is strongly limiting mass transport. However, because the ratio is > 1, we have that the transport is slightly gas phase controlled.

Question 2 – Solutions (9 points)

a) (5.5 pts) Let's consider three points, point 1 at the top of the water in the tower, point 2 at point A, and point 3 at point B.



We begin by applying Bernoulli's equation between points 1 and 2. We can neglect friction losses and there is no pump so:

$$P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

$$P_2 = P_1 + \rho g(h_1 - h_2) + \frac{\rho(v_1^2 - v_2^2)}{2}$$

We can assume that the speed of the water at the top of the tower is negligibly small by the continuity equation. Since $d_1 >> d_2$, we will have that $v_2 >> v_1$.

Indeed, the continuity equation at point 1 gives:

$$v_1 = \frac{Q}{A_1} = \frac{2{\cdot}10^{-3} m/s}{\frac{\pi(4m)^2}{4}} = 1.59{\cdot}10^{-4}\frac{m}{s} \approx 0$$

So, it is fine to use $v_1 = 0$.

Similarly, we can calculate v_2.

$$v_2 = \frac{Q}{A_2} = \frac{2{\cdot}10^{-3} m/s}{\frac{\pi(0.05m)^2}{4}} = 1.02\frac{m}{s}$$

Therefore:

$$P_2 = P_1 + \rho g(h_1 - h_2) - \frac{\rho v_2^2}{2}$$

$$P_2 = 101'325\,[Pa] + 1000\frac{kg}{m^3}{\cdot}9.81\frac{m}{s^2}(30 - 0)\,[m] - \frac{1000\frac{kg}{m^3}\left(1.02\frac{m}{s}\right)^2}{2} = 395'105[Pa]$$

We can now apply Bernoulli's equation between points 2 and 3. We can neglect friction losses and there is no pump so:

$$P_2 + \rho g h_2 + \frac{\rho v_2^2}{2} = P_3 + \rho g h_3 + \frac{\rho v_3^2}{2}$$

$$P_3 = P_2 + \rho g(h_2 - h_3) + \frac{\rho(v_2^2 - v_3^2)}{2}$$

By using the continuity equation:

$$v_3 = \frac{Q}{A_3} = \frac{2{\cdot}10^{-3} m/s}{\frac{\pi(0.025m)^2}{4}} = 4.08\frac{m}{s}$$

So

$$P_3 = 395'105[Pa] + 1000\frac{kg}{m^3}\cdot 9.81\frac{m}{s^2}(0-5)\ [m] + \frac{1000\frac{kg}{m^3}\left[(1.02\frac{m}{s})^2 - (4.08\frac{m}{s})^2\right]}{2}$$

$$P_3 = 338'252[Pa]$$

b) (3.5 pts) We have now that:

$$P_{3,new} = 6\ \ [bar]\ = 600'000\,[Pa]$$

The Bernoulli equation between point 2 that we chose before the pump and point 3 is now:

$$P_2 + \rho g h_2 + \frac{\rho v_2^2}{2} + P_{pump} = P_{3,new} + \rho g h_3 + \frac{\rho v_3^2}{2}$$

We have now:

$$P_{3,new} = P_2 + \rho g(h_2 - h_3) + \frac{\rho(v_2^2 - v_3^2)}{2} + P_{pump}$$

From a), we have:

$$P_{3,new} = 600'000\,[Pa]\ = 338'252[Pa] + P_{pump}$$

So:

$$P_{pump} = 261'748\,[Pa]$$

The pump power is:

$$Power = Q\rho g H_p = QP_{pump} = 2\cdot 10^{-3}\left[\frac{m}{s}\right] \times 261'748\,[Pa]\ = 523.5\ \ [W]$$

If the mechanical efficiency of the pump is 80% ($\eta = 80\%$), then the mechanical power required to operate the pump is equal to:

$$Mechanical\ Power = \frac{Power}{\eta} = 654.4\ [W]$$

Question 3 – Solution (14 points)

1) (3pts) If there is no insulation layer, heat transfer occurs only through external convection. Therefore, the heat transfer from the air is:

$$\dot{Q} = A_{convec.}h(T_{air} - T_{N2})$$

The surface area for convection is the external surface area of a sphere:

$$A_{convec.} = 4\pi r^2 =\ 4\pi(1.5\ m)^2 = 28.27\ m^2$$

We know $T_{air} = 15\ \ ^oC\ and\ T_{N2} = -196\ \ ^oC$ .Therefore:

$$\dot{Q} = Ah\left(T_{air} - T_{N2}\right) = 28.27\ \left[m^2\right] \cdot 35\ \left[\frac{W}{m^2 \cdot K}\right] \cdot (15 - (-196))\,[K] = 208'773\ W$$

$$= 208.773\ kJ \cdot s^{-1}$$

The evaporation rate of liquid nitrogen is given by: ($L_{vap} = 198\ kJ/kg$)

$$\dot{Q} = \dot{m} \cdot L_{vap}$$

Therefore:

$$\dot{m} = \frac{\dot{Q}}{h_{vaporization}} = \frac{208.773\ W}{198\ kJ \cdot kg^{-1}} = 1.055\ kg \cdot s^{-1}$$

2) (3pts) The evaporation rate is 1.055 kg/s from 1).

The time required to completely evaporate nitrogen is given by:

$$t = \frac{m}{\dot{m}}$$

The total mass of nitrogen is simply given by:

$$m = V \cdot \rho = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi\,(1.5m)^3 \cdot 807\frac{kg}{m^3} = 11'409\ \ [kg]$$

Therefore:

$$t = \frac{11'409\ \ [kg]}{1.055\ [kg \cdot s^{-1}]} = 10'814\,[s]\ = 180\ \ [min]\ = 3[h]$$

3) (2pts) To derive the thermal resistance for conduction in a spherical geometry, we need to solve the Fourier's law differential equation:

$$\dot{Q} = -kA\frac{dT}{dr}$$

For a spherical geometry, the area (A) is $4\pi r^2$. Therefore, the equation becomes

$$\dot{Q} = -4k\pi r^2\frac{dT}{dr}$$

Rearranging the equation yields

$$\dot{Q} \cdot \frac{-1}{4k\pi} \cdot \int_{r_1}^{r_2} \frac{dr}{r^2} = \int_{T_1}^{T_2} dT$$

The resistance can be calculated by solving the equation above. By definition, the thermal resistance will be:

$$\dot{Q} = \frac{\Delta T}{R_{cond}}$$

4) (5.5 pts) With the insulation layer, there are two heat transfer resistance in the system: external convection of air, and conduction inside the insulating layer. These two thermal resistances are in series. Therefore,

$$R_{eq} = R_{convection} + R_{conduction}$$

First, let's calculate $R_{convection}$:

Note that due to the addition of the insulation layer, the area of convection is slightly bigger.

$$A_{convec.} = 4\pi r^2 = 4\pi(1.55\ m)^2 = 30.2\ m^2$$

$$R_{convection} = \frac{1}{Ah} = \frac{1}{30.2[m^2]\cdot 35\left[\frac{W}{m^2\cdot K}\right]} = 9.46\cdot 10^{-4} K\cdot W^{-1}$$

We can calculate $R_{convection}$ with the given formula:

$$R_{conduction} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4k\pi} = \frac{\left(\frac{1}{1.5m} - \frac{1}{1.55m}\right)}{4\pi\cdot\left(0.035\ \frac{W}{m\cdot K}\right)} = 0.0489\ K\cdot W^{-1}$$

Hence:

$$R_{eq} = R_{convection} + R_{conduction} = 9.46\cdot 10^{-4} + 0.0489 = 0.0498\ K\cdot W^{-1}$$

The heat transferred to the nitrogen is then:

$$\dot{Q} = \frac{(T_{air} - T_{N2})}{R_{eq}} = \frac{(15 - (-196))\,[K]}{0.0498\left[\frac{K}{W}\right]} = 4237\ W = 4.237\ (kJ\cdot s^{-1})$$

The external temperature of the insultation layer in contact with air can be found with one of the equations below:

$$\dot{Q} = \frac{(T_{insulation} - T_{N_2})}{R_{conduction}}$$

$$\dot{Q} = \frac{(T_{insulation} - T_{air})}{R_{convecion}}$$

So:

$$T_{insulation} = T_{air} - Q\cdot R_{convection} = 15°C - 4237\ W\cdot 9.46\cdot 10^{-4}\ \left[K\cdot W^{-1}\right] = 11°C$$

$$T_{insulation} = T_{N_2} + Q\cdot R_{conductino} = -196°C + 4237\ W\cdot 0.0489\ \left[K\cdot W^{-1}\right] = 11°C$$

The rate of evaporation is:

$$\dot{Q} = \dot{m} \cdot L_{vap}$$

$$\dot{m} = \frac{\dot{Q}}{L_{vap}} = \frac{4.233\ kJ \cdot s^{-1}}{198\ kJ/kg} = 0.0214\ kg/s$$

5) (0.5 pts) This time, we have:

$$t = \frac{m}{\dot{m}} = \frac{11'409\ [kg]}{0.0214\ [kg \cdot s^{-1}]} = 533'131\ [s] = 8'886\ [min] = 148\ [h] = 6.2[days]$$

Question 4 – Solution (11 points)

1) (2.5 pts) At steady-state the molar flux of CO_2 (A) inside the membrane (B) is given by Fick's law:

$$j_{CO2} = -\frac{D_{AB}\left(C_{CO2,0,P} - C_{CO2,0,F}\right)}{L}$$

We know:

- $D_{AB} = 1.28 \bullet 10^{-8}\ cm^2/s = 1.28 \bullet 10^{-12}\ m^2/s$ for CO_2 (A) in the membrane (B)
- $L = 5\ \mu m = 5 \cdot 10^{-6}$ m
- $C_{CO2,bulk,P} = 0\ mol/m^3$ because CO_2 is constantly being removed by N_2 on the permeant side, thus $C_{CO2,0,P} = 0\ mol/m^3$
- $C_{CO2,0,F} = 40\ C_{CO2,bulk,F}$ on the feed side

The concentration of CO_2 in the gas phase is given by:

$$C_{CO2,bulk,F} = \frac{P_{CO2,bulk,F}}{RT} = \frac{10'000\ [Pa]}{8.314\ \left[\frac{J}{mol \cdot K}\right] \cdot 298.15\ [K]} = 4.03\left[\frac{mol}{m^3}\right]$$

Therefore:

$$C_{CO2,0,F} = 40 \cdot C_{CO2,bulk,F} = 40 \cdot 4.03\left[\frac{mol}{m^3}\right] = 161\left[\frac{mol}{m^3}\right]$$

The flux is:

$$j_A = -\frac{1.28 \cdot 10^{-12}\left[\frac{m^2}{s}\right]\left(0 - 161\left[\frac{mol}{m^3}\right]\right)}{5 \cdot 10^{-6}\ [m]} = 4.12 \cdot 10^{-5}\left[\frac{mol}{m^2 s}\right]$$

2) (2.5 pts) With the information given, the flux of CH4 (C) through the membrane (B) is:

$$j_{CH4} = \frac{1}{100} j_{CO2} = -\frac{D_{CB}\left(C_{CH4,0,P} - C_{CH4,0,F}\right)}{L} = 4.12 \cdot 10^{-7}\left[\frac{mol}{m^2 s}\right]$$

Where:

- $L = 5\ \mu m = 5 \cdot 10^{-6}$ m

- $C_{CH4,bulk,P}$ = 0 mol/m^3 because CH_4 is constantly being removed by N_2 in the bulk of the permeant side, thus $C_{CH4,0,P}$ = 0 mol/m^3
- $C_{CH4,0,F}$ = 6 $C_{CH4,bulk,F}$ on the feed side

Similarly to the case with CO_2:

$$C_{CH4,0,F} = 6 \cdot C_{CH4,bulk,F} = 6 \cdot 4.03 \left[\frac{mol}{m^3}\right] = 24.2 \left[\frac{mol}{m^3}\right]$$

Thus, we have:

$$D_{CB} = \frac{-L}{100\left(C_{C,0,P} - C_{C,0,F}\right)} j_A$$

$$D_{CB} = \frac{-5 \cdot 10^{-6}\,[m]}{100\left(0 - 24.2\left[\frac{mol}{m^3}\right]\right)} \cdot 4.12 \cdot 10^{-5} \left[\frac{mol}{m^2 s}\right] = 8.51 \cdot 10^{-14} \left[\frac{m^2}{s}\right]$$

3) (6 pts) To calculate the molar flux rate in the membrane we need to find the concentration of CH_4 and CO_2 at the interface on the permeant side. We can calculate the bulk concentration of CH_4 and CO_2 at the permeant side from ideal gas relation (T=298 K).

First, we calculate the partial pressure of CO_2 and CH_4 **on the permeant side** from the given information.

$$P_{CO2,bulk,P} = 5000\ Pa\ \times 0.9 = 4500\ [Pa]$$

$$P_{CH4,bulk,P} = 5000\ Pa\ \times 0.1 = 500\ [Pa]$$

Using ideal gas relation, **bulk** concentration of CO_2 and CH_4 are calculated as:

$$P_{CO2,bulk,P} = C_{CO2,bulk,P}RT = 4500\ [Pa] = C_{CO2,bulk,P} \times 8.314\ \left[\frac{Pa \cdot m^3}{mol \cdot K}\right] \times 298\ [K]$$

$$C_{CO2,bulk,P} = 1.8 \left[\frac{mol}{m^3}\right]$$

Similarly,

$$P_{CH4,bulk,P} = C_{CH4,bulk,P}RT = 500\ [Pa] = C_{CH4,bulk,P} \times 8.314\ \left[\frac{Pa \cdot m^3}{mol \cdot K}\right] \times 298\ [K]$$

$$C_{CH4,\ bulk,P} = 0.2 \left[\frac{mol}{m^3}\right]$$

We know CO_2 and CH_4 in the bulk are in equilibrium with the membrane interface at the permeant side. We know that at equilibrium $C_{CO2,\ 0,P} = 40\ C_{CO2,bulk,P}$ *and* $C_{CH4,\ 0,P} = 6\ C_{CH4,bulk,P}$.

With this information we can calculate $C_{CO2,\ 0,\ P}$ and $C_{CH4,0,\ P}$:

$$C_{CO2,\ 0,\ P} = 72\ \left[\frac{mol}{m^3}\right]$$

$$C_{CH4,\ 0,\ P} = 1.2 \left[\frac{mol}{m^3}\right]$$

The interfacial concentration of CO_2 and CH_4 on the feed side does not change, and we can use the values calculated in part 1) and 2). Therefore, the molar flux is calculated as:

$$j_{CO2} = -\frac{D_{AB}\left(C_{CO2,0,P} - C_{CO2,0,F}\right)}{L} = \frac{1.28 \cdot 10^{-12}\left[\frac{m^2}{s}\right]\left(72 - 161\left[\frac{mol}{m^3}\right]\right)}{5 \cdot 10^{-6}\,[m]}$$

$$j_{CO2} = 2.3\ \times 10^{-5}\ \left[\frac{mol}{m^2 s}\right]$$

Similarly,

$$j_{CH4} = -\frac{D_{CB}\left(C_{CH4,0,P} - C_{CH4,0,F}\right)}{L} = \frac{8.51 \cdot 10^{-14}\left[\frac{m^2}{s}\right]\left(1.2 - 24.2\left[\frac{mol}{m^3}\right]\right)}{5 \cdot 10^{-6}\,[m]}$$

$$j_{CH4} = 3.9\ \times 10^{-7}\left[\frac{mol}{m^2 s}\right]$$

Question 5 – Solution (7 points)

a) (2.5 pts) We know:

- $\dot{m}_{oil} = 0.4\left[\frac{kg}{s}\right] and\ \dot{m}_{water} = 0.2\left[\frac{kg}{s}\right]$
- $T_{h,i} = 60\ \ ^{o}C,\ T_{h,o} = 40\ ^{o}C,$
- $T_{c,i} = 30\ \ ^{o}C$
- $C_{p,\,oil} = 2006\ \left[\frac{J}{kg.K}\right], C_{p,water} =\ 4178\ \left[\frac{J}{kg.K}\right]$

We want to calculate:

- $T_{c,o} = ?and\ Q = ?$

To calculate the heat transferred between the two fluids we can use their heat capacity and their change in temperature:

For the oil, we have:

$$Q_h = \dot{m}_{(oil)} C_{p(oil)} \Delta T_{(oil)}$$

$$Q_h = 0.4\ \left[\frac{kg}{s}\right] \times 2006\ \left[\frac{J}{kg \cdot K}\right] \times (60 - 40)\ [K]\ = 16{,}048\ [W]$$

By using the same equation for the water, we can find $T_{c,o}$.

And

$$Q_h = Q_c$$

So

$$Q_c = \dot{m}_{(water)} C_{p(\ water)} \Delta T_{(water)}$$

$$T_{c,o} = T_{c,i} + \frac{Q_c}{\dot{m}_{(water)} \cdot C_{p(\ water)}} = 30°C + \frac{16{,}048\ [W]}{0.2\ \left[\frac{kg}{s}\right] \cdot 4178\ \left[\frac{J}{kg \cdot K}\right]} = 49.2\ [\ {}^{o}C]$$

b) (2.5 pts) We can calculate DT$_{lm}$ by using the equation for counter-flow heat exchanger:

$$\Delta \mathrm{T}_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Where $\Delta T_1 = T_{h,i} - T_{c,o}$ and $\Delta T_2 = T_{h,o} - T_{c,i}$

$$\Delta \mathrm{T}_{lm} = \frac{(60 - 49.2) - (40 - 30)}{\ln\left(\frac{60 - 49.2}{40 - 30}\right)} = 10.4\ [\ {}^{o}C]$$

We can find the overall heat transfer coefficient (U) by applying the equation $Q = A \times U \times \Delta \mathrm{T}_{lm}$.

$$Q = 16{,}048\ \ [W] = 6.8\ \left[m^2\right] \times U \times 10.4\ [\ {}^{o}C]$$

$$U = 227\ [\frac{W}{K.m^2}]$$

c) (2 pts) To take into account the deposition of salts and the fouling factor, we have to recalculate U_{new} based on the below equation:

$$\frac{1}{U_{new}} = R_{Convection} + R_{condution} + R_{convection} + R_{fouling}$$

We know from previous section that:

$$\frac{1}{U} = R_{Convection} + R_{condution} + R_{convection} = \frac{1}{227}\ [\frac{K.m^2}{W}]$$

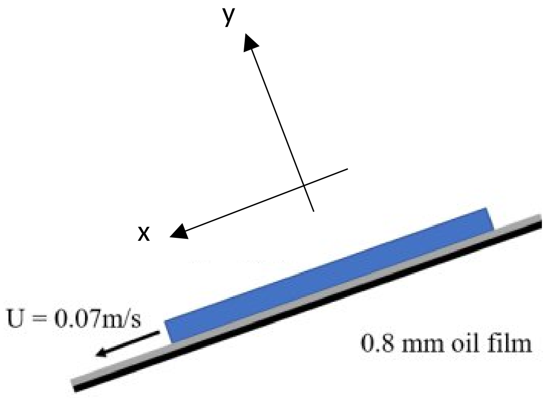
By inserting this information and $R_{fouling}$:

$$\frac{1}{U_{new}} = \frac{1}{227}\ \left[\frac{K.m^2}{W}\right] + 0.000176 \left[\frac{K.m^2}{W}\right]$$

$$U_{new} = 218.3\ [\frac{W}{K.m^2}]$$

Question 6 – Solution (4 points)

Let's consider the following coordinate system.



The drag force caused by the viscosity of the oil must equal the component of weight along the surface by Newton 1st law since velocity is constant (i.e. no acceleration)

$$\sum\vec{F} = m\vec{a} = \vec{F}_{gravity} + \vec{F}_{shear} = 0$$

So:

$$\vec{F}_{gravity} = -\vec{F}_{shear}$$

Let's consider the projections in the x-direction:

$$F_{gravity,x} = -F_{shear,x} = -A\cdot\tau_{xy}$$

$$m\cdot g\cdot\sin(\theta) = -\left(-A\cdot\mu\frac{dv_x}{dy}\right) = A\cdot\mu\frac{v_x}{h}$$

So:

$$\mu = \frac{m\cdot g\cdot\sin(\theta)\cdot h}{v_x\cdot A}$$

$$\mu = \frac{5.1\ kg\cdot 9.81\ \frac{m}{s^2}\cdot\sin(20)\cdot 0.0008\ m}{0.07\ \frac{m}{s}\cdot(0.5m)^2} = 0.782\ \left[\frac{kg}{m\cdot s} = Pa\cdot s\right]$$